Lunar Laser Ranging and the Evolution of Lunar Dynamics

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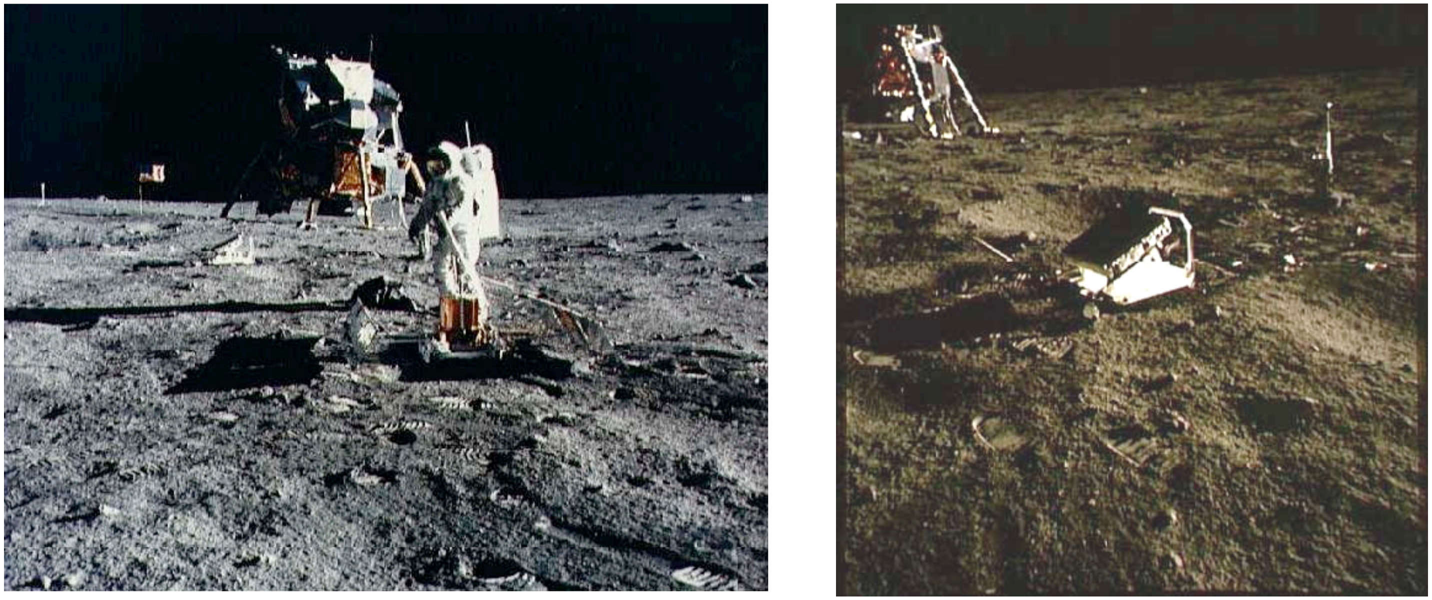
Introduction — the Start



- The July 1969 placement of the Apollo 11 laser retroreflector initiated a shift from analyzing lunar position angles to ranges.

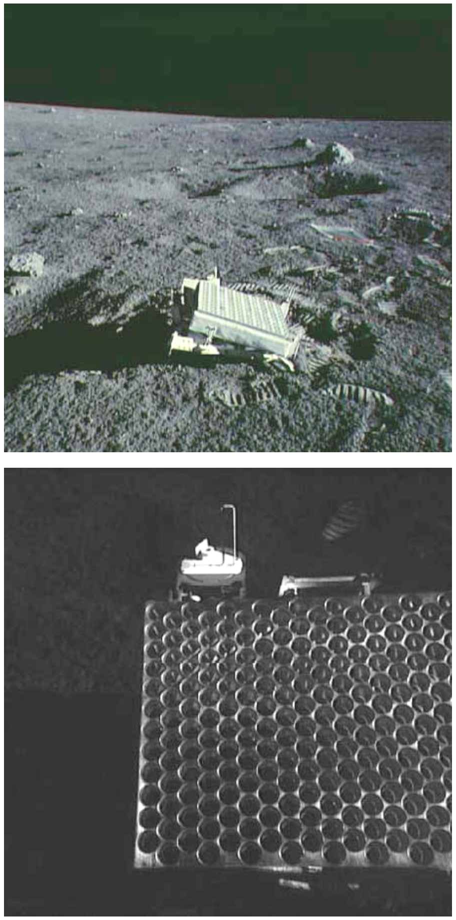
Introduction

- Range data required a different analysis approach.
- Lunar Laser Ranging (LLR) has stimulated the development of lunar dynamics and modeling for 37 yr.
- This talk will combine dynamics and modeling with some history mixed in.

Lunar Laser Ranges

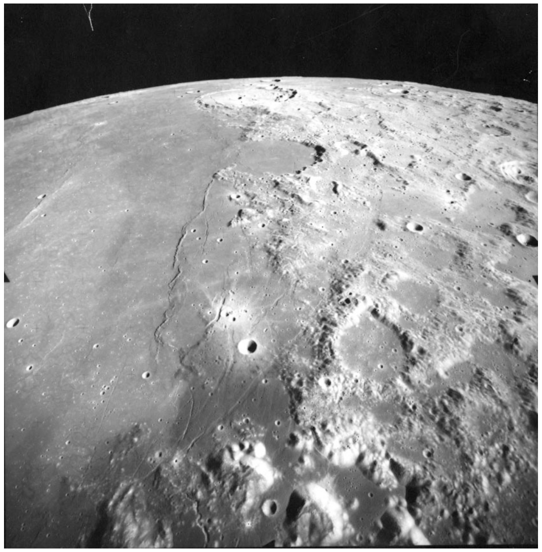
- Laser pulses are sent from stations on the Earth toward the Moon where they bounce off of retroreflector arrays and return to the Earth.
- Ranging started in 1969 and continues to present.

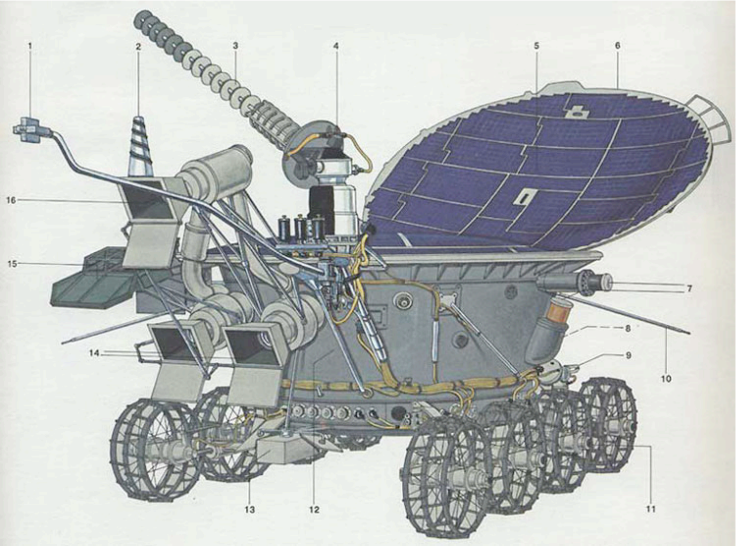
Apollo 14 & 15 Retroreflectors



- Apollo 14 February 1971
- Apollo 15 July 1971

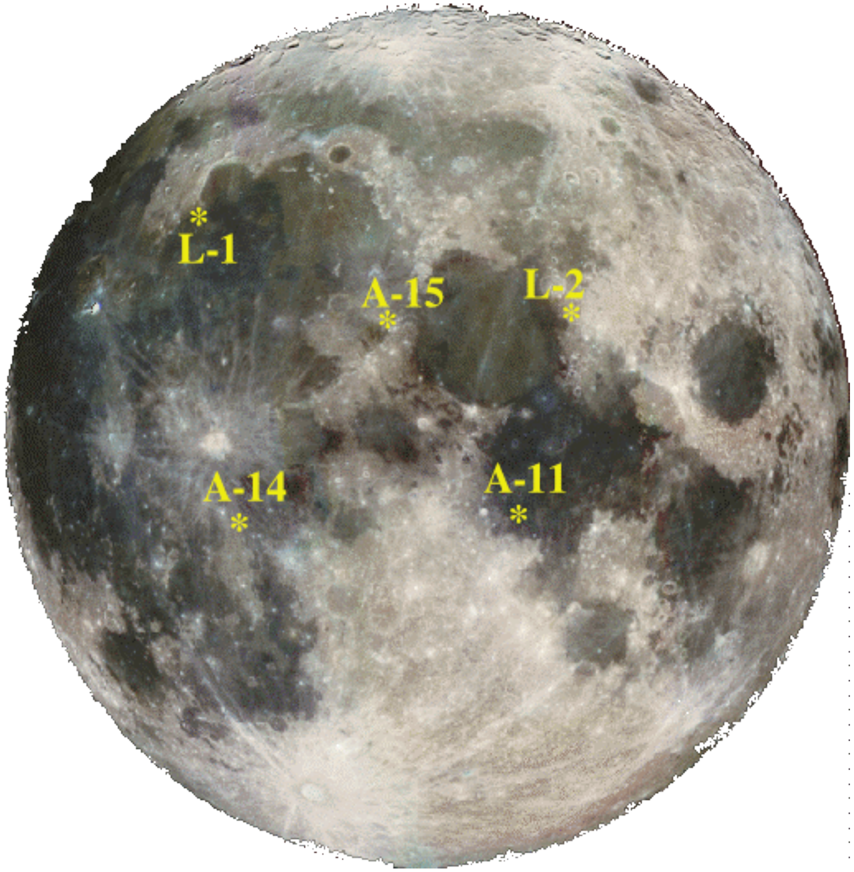
Lunokhod 2





- Luna 21 landed January 1973 in the crater Le Monnier which cuts into edge of Mare Serenitatis.
- The Lunokhod 2 rover traveled 37 km.
- French-built retroreflector sticks out at top left.

Retroreflector Locations



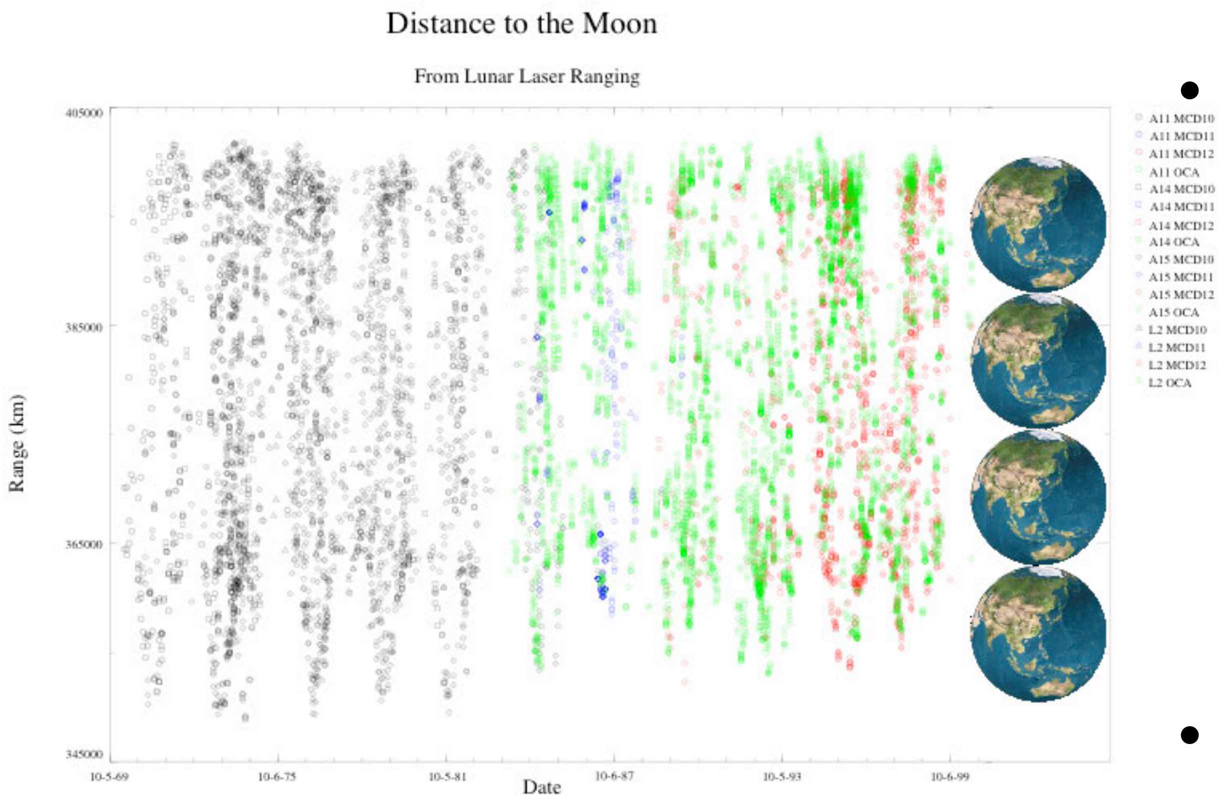
Early History

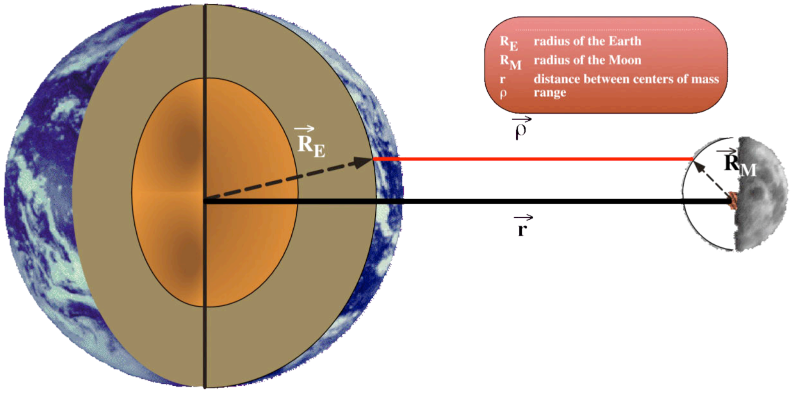
- The Lunar Laser Ranging effort was conceived and proposed as an Apollo experiment in the 1960s.
- The first ranges were in 1969, shortly after the Apollo 11 landing.
- I became involved with the analysis in 1971.

Goals in 1969 Apollo 11 Report

- Study of gravitation and relativity (secular variation in the gravitational constant),
- the physics of the Earth (fluctuation in rotation rate, motion of the pole, large-scale crustal motions),
- the physics of the Moon (physical librations, center-of-mass motion, size and shape).

Raw Ranges

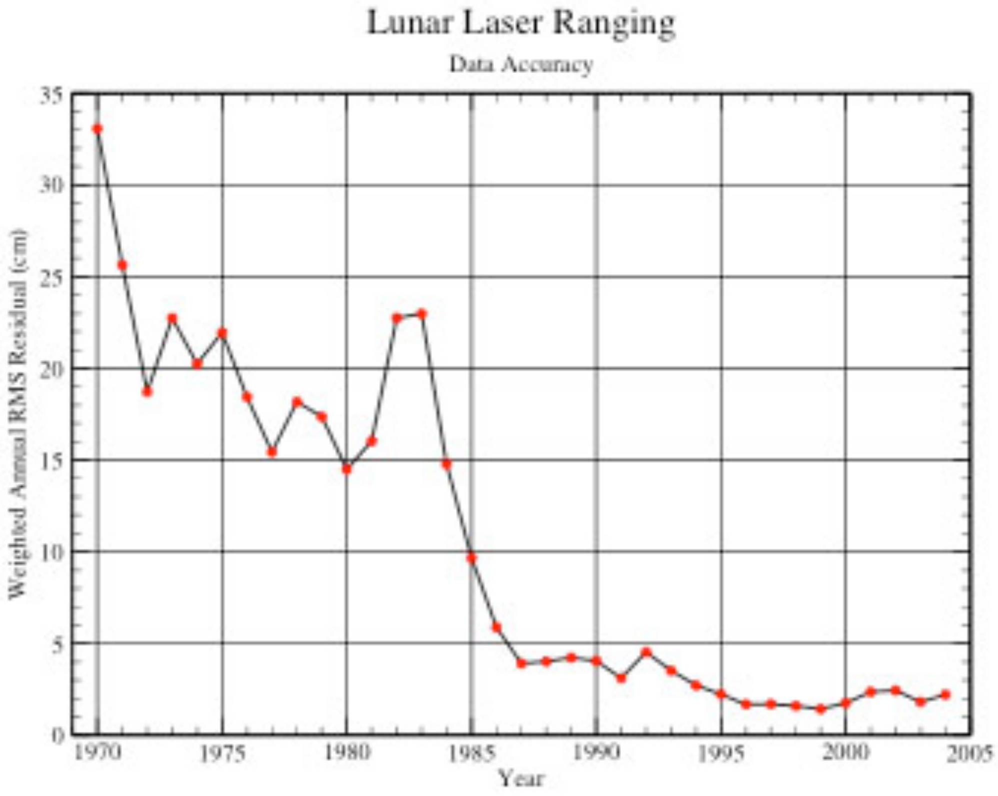




R_E radius of the Earth
R_M radius of the Moon
r distance between centers of mass
ρ range

- Station to retroreflector distances. Earth shows scale. Colors are different station/reflector pairs.
- Ranges are available at International Laser Ranging Service website.

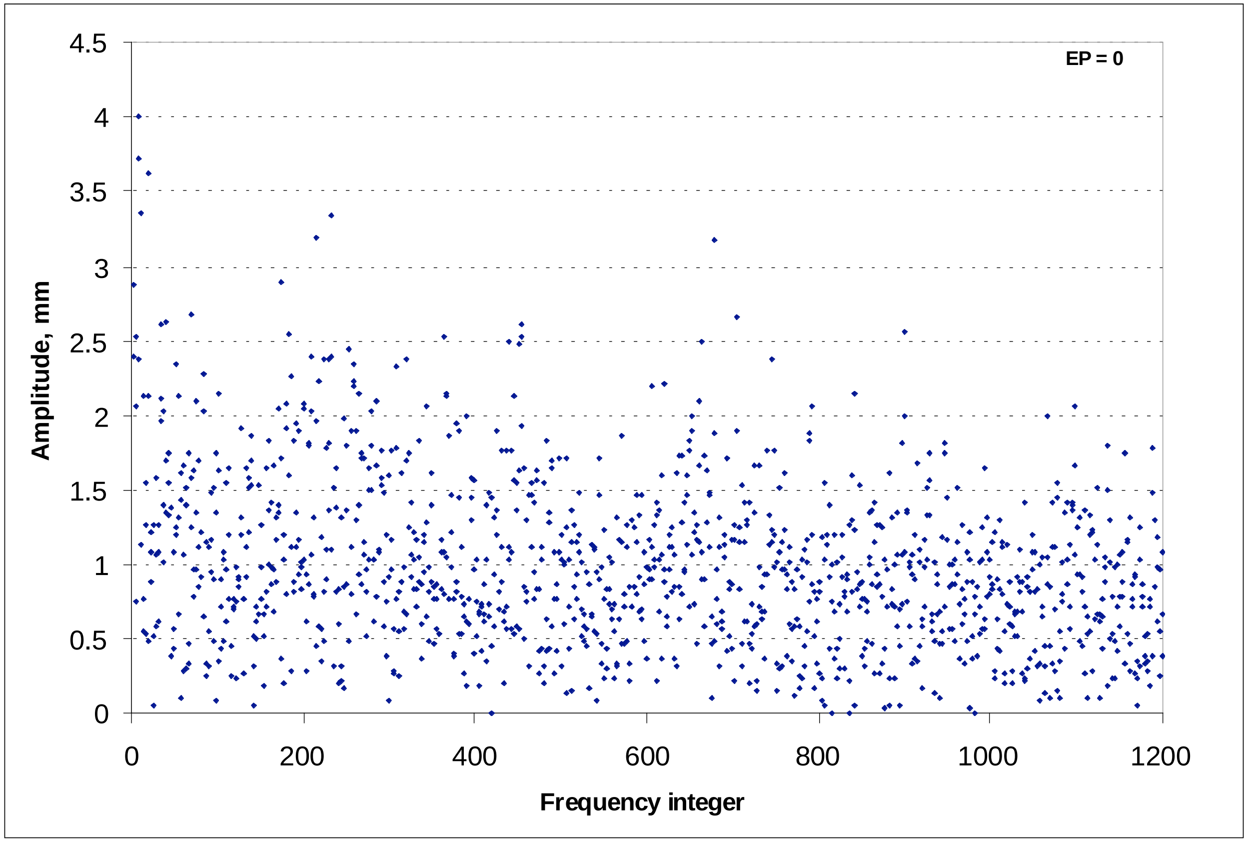
RMS Residual vs. Year



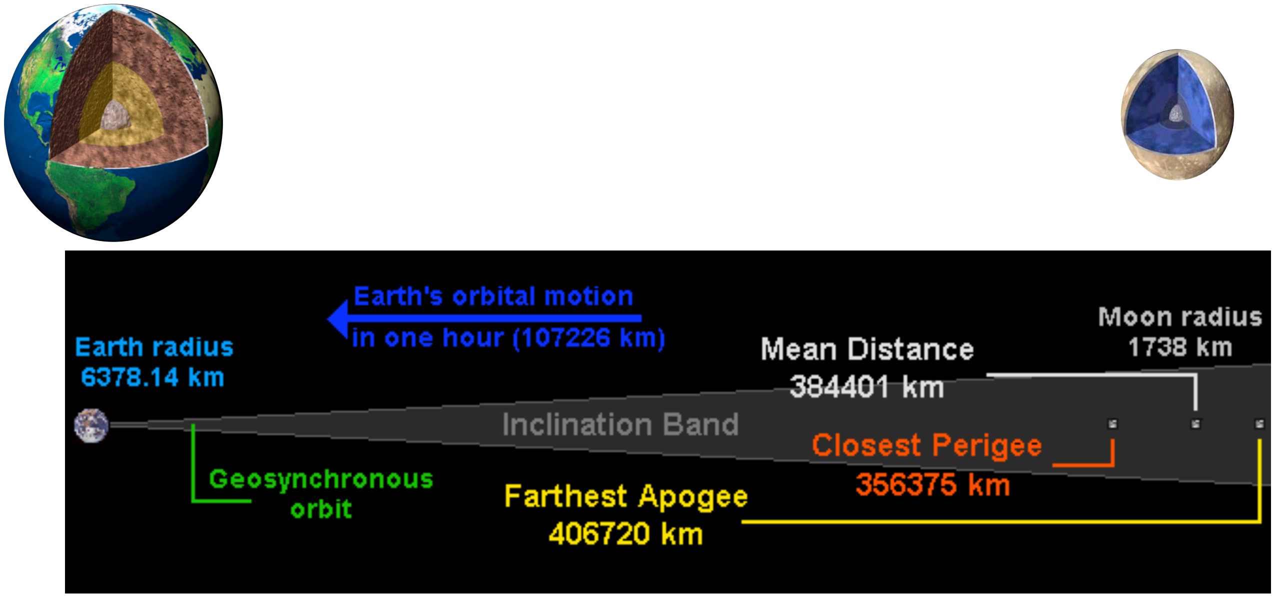
Data Fitting

- Starting with ~50,000 km variations and ending up with few centimeter residuals is done with detailed modeling of the range and weighted least-squares fits.
- Spectrum of residuals has a 4 mm maximum and a 1 mm background.

Spectrum of Weighted Residuals



Earth - Orbit - Moon



- Earth radius 6371 km
- Orbit semimajor axis 384,399 km
- Moon equatorial radius 1738 km
- Ratio 1:60:1/4

Analysis Concept

- For the analysis of angular data, the orbit is the main concern
- For range data, the center-to-center orbit is only part of the problem. The geocentric ranging station location and the Moon centered retroreflector position must be determined.

Dynamical Computations

- Joint numerical integration of the orbits of the Moon, Earth, and planets plus lunar rotation.
- Model includes relativistic Earth-Moon-planet interactions, gravitational harmonic coefficients for Earth (zonal), Moon and Sun (J_2), tides on Earth and Moon, and a fluid lunar core.

Dynamical Partials

- Numerical integration of partial derivatives of the orbits and lunar Euler angles with respect to solution parameters such as initial conditions, mass ratios, gravity coefficients, and tide, core, and relativity parameters.

Mean Lunar Orbit

- Semimajor axis 384,399 km
- Eccentricity 0.0549
- Inclination 5.145˚
- Sidereal period 27.322 days
- Anomalistic period 27.555 days
- Nodical period 27.212 days

Perturbed Orbit

- Radius series from Chapront-Touzé and Chapront 385,001 – 20,905 cos l – 3699 cos(2D- l) – 2956 cos 2D – 570 cos 2l…km
- Mean anomaly l has a 27.555 day period..
- D is mean elongation from Sun with a 29.531 day period.
- Two solar perturbation terms, arguments with D, are stronger than the e^2 (2l) term.

Largest Radial Amplitudes by Cause

Cause	Amplitude
–Ellipticity	20905 & 570 km
–Solar perturbations	3699 & 2956 km
–Jupiter perturbation	1.06 km
–Venus perturbations	0.73, 0.68 & 0.60 km
–Earth J_2	0.46 & 0.45 km
–Moon J_2 & C_{22}	0.2 m
–Earth C_{22}	0.5 mm
–Solar radiation pressure	4 mm

Relativistic Effects on Orbit

Cause	Amplitude
– Lorentz contraction	0.95 m
– Solar potential	6 cm
– Time transformation	5 & 5 cm
– Other relativity	5 cm

Sources: Chapront-Touzé and Chapront, Vokrouhlicky, Williams and Dickey

Causes of Perigee and Node Precessions

Cause	ϖ rate "/yr	Ω rate "/yr
Sun	146,425.38	–69,671.67
Planets	2.47	–1.44
Earth J_2	6.33	–5.93
Moon J_2 & C_{22}	–0.0176	–0.1705
Relativity	0.0180	0.0190

Orbit — Tidal Dissipation

- Semimajor axis 37.9 mm/yr
- Tidal acceleration –25.7 “/cent^2
- Both Earth and Moon have tidal dissipation. From the Earth –26.0 “/cent^2 while from the Moon +0.3 “/cent^2.
- Artificial satellite tide results predict –25 “/cent^2, the difference seems significant, and the discrepancy is not understood.

Tidal Acceleration — History

- The 1939 Spencer Jones value of –22.4 “/cent^2 had to separate acceleration of Earth rotation.
- Several 1970s values used the short span after atomic time became available. These values were much larger (–36, –38, etc.) and discordant.
- Including Mercury transits with older lunar data, Morrison and Ward (1975) got –26±2 “/cent^2.
- Two LLR papers in 1978 got –23.8±4 and –24.6±5 “/cent^2 with data using atomic time while agreeing with results taking time from dynamics.

Orbit — Eccentricity Rate

• Tides on Earth	$1.3\mathrm{x}10^{-11}$ /yr
• Tides on Moon	$-0.6\mathrm{x}10^{-11}$ /yr
• Anomalous rate	$(1.6\pm0.4)\mathrm{x}10^{-11}$ /yr
• Total	$2.3\mathrm{x}10^{-11}$ /yr

- The anomalous rate amounts to 6 mm/yr in perigee and apogee distance and the cause is unknown.

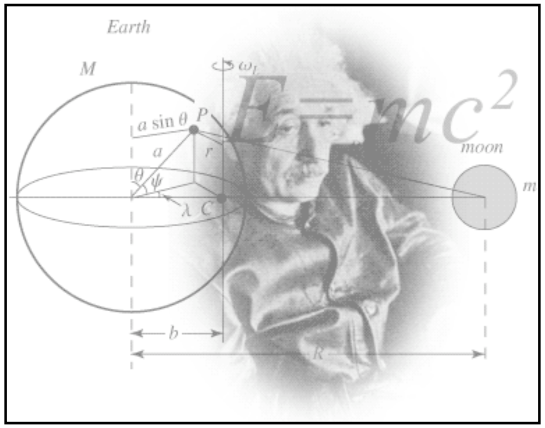
Orbit — Gravitational Physics

- The equivalence principle test is sensitive to the gravitational/inertial mass ratio difference between the Earth and Moon.
  $$(-1.0 \pm 1.4) \times 10^{-13}$$
- Equivalent to amplitude of 3±4 mm at 29.53 days.
- PPN beta–1 is $(1.2 \pm 1.1) \times 10^{-4}$
- Geodetic precession confirmed with 0.64% (0.00012 “/yr) uncertainty compared to its value of 0.0192”/yr.

More Gravitational Physics

- Relative rate for gravitational constant.

  $(4 \pm 9) \times 10^{-13}$ /yr
- Results in accord with general relativity. The solar system does not share cosmic expansion.



Earth Effects

- Station location: position, plate motion, solid-body tides
- Earth orientation: precession, nutation, UT1, and polar motion
- Relativistic time and position effects
- Atmospheric delay

Earth

- The model for the Earth has become more complex through the years. Major complexities come from the oceans, atmosphere and ground water. These variations cause small effects on tides, loading, nutation, UT1 and polar motion.

Status of LLR Stations

- McDonald, Texas — operating 37 yr
- Grasse, France (OCA) — being upgraded after 2+ decades of operation
- Apache Point, New Mexico — new station has first ranges, being debugged
- Matera, Italy — first two days of data
- South Africa — installing former OCA LLR

McDonald and Grasse

McDonald, Texas

Grasse, France

Station Motion

		Solution mm/yr	Model mm/yr
McD	East	–13.1±2.5	–12.7
	North	–8.0±3.3	–6.2
	Up	–0.8±1.8	0.8
OCA	East	19.5±1.4	20.7
	North	17.5±2.3	15.9
	Up	4.1±2.0	1.0

LLR Geophysics & Geodesy

- The main limitation to LLR results for the Earth is the small number (2) of stations operating during the last two decades.
- For those two sites there are station position, motion, UT0 and variation of latitude results.
- Can also determine precession & obliquity rates, 18.6 yr nutation, orientation, and diurnal & semidiurnal UT1 variations.

The Earth Seen from the Moon?

- If an astronaut watched the motion of the Earth in the lunar sky, what would be seen?

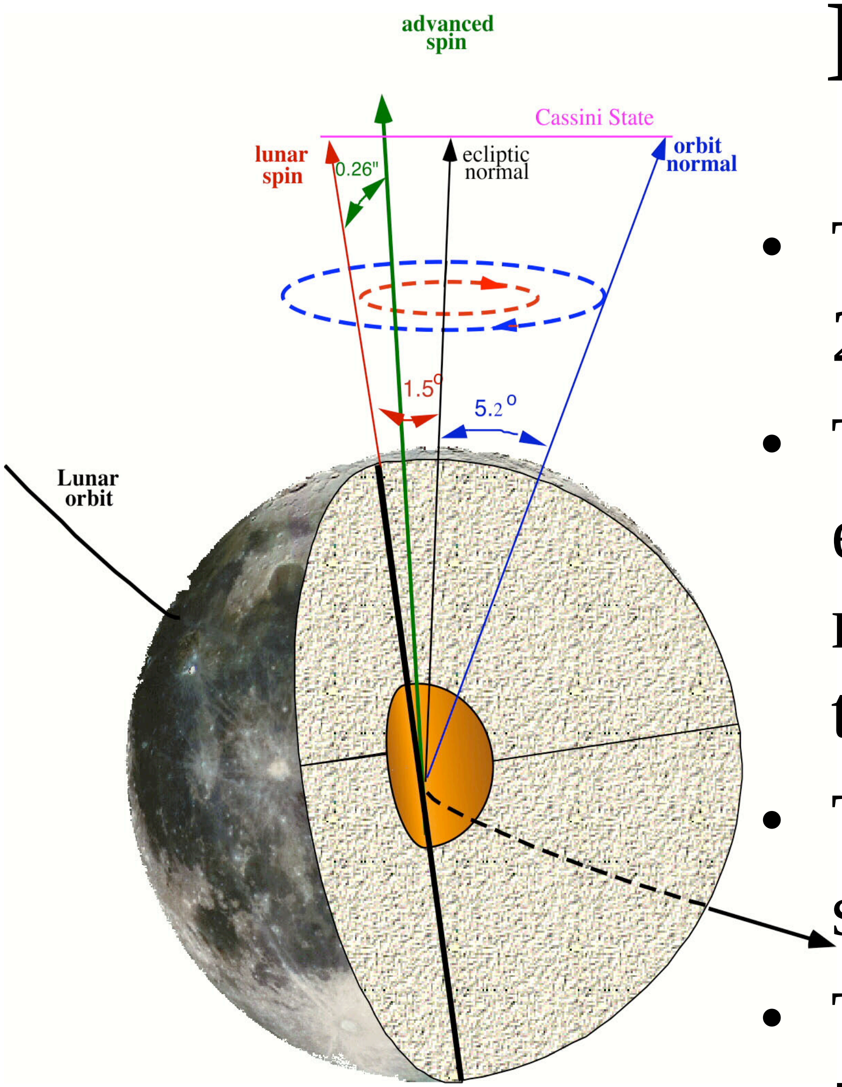
Earth Seen from Moon

- The Earth oscillates in longitude and latitude. The main monthly longitude amplitude is 6.3˚ while the monthly latitude amplitude is 6.7˚. The two main periods beat with the 6.0 yr argument of perigee period. The path varies from nearly circular, through a tilted ellipse, to a tilted line. Smaller terms modify the pattern.

Solid-Body Tides on the Moon

- The apparent motion of the Earth in longitude and latitude plus the distance variation causes tidal variations.
- The two major tidal terms are the two monthly terms with the 6.0 yr beat period.
- The vertical amplitudes are up to 9 cm, depending on location.

Lunar Cassini State



- The sidereal rotation period is 27.322 days.
- The descending node of the equator on the ecliptic plane matches the ascending node of the orbit.
- The nodes regress with the same 18.6 yr period.
- The total angle i+I is 5.145˚+1.543˚=6.69˚.

Moon Effects

- Retroreflector location: position, solid-body tides, relativistic effects.
- Moon orientation, physical librations, from numerical integration.

Physical Librations — Early

- Series representation from D. H. Eckhardt.
- Two solution parameters: (C–A)/B and (B–A)/C. A, B & C are the principal moments of inertia.

Physical Librations — Now

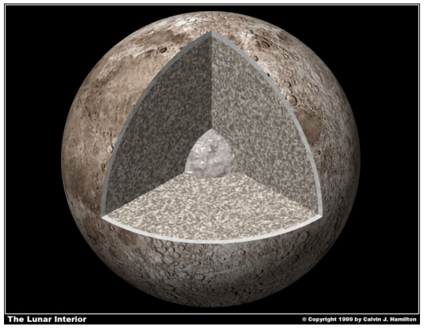
- Numerically integrated Euler angles. Torques from Earth, Sun, 3 planets, harmonics degree 2-4, figure-figure, tides, & fluid core.
- Possible solution parameters: (C–A)/B and (B–A)/C, J_2, 7 third-degree coefficients, tidal Love number k_2 & time delay (dissipation), solid-mantle/liquid-core boundary dissipation and oblateness, core moment, dissipation coefficients, initial conditions for mantle and core.

Lunar Geometrical Effects

- Solve for 3 coordinates for each of 4 retroreflectors. Uncertainty <1 m.
- Can fix or solve for tidal displacement Love numbers h_2 and l_2.
- Relativistic coordinate transformations are modeled.

Lunar Solid-Body Tides

- $k_2 = 0.0205 \pm 0.0025$, $h_2 = 0.041 \pm 0.009$, l_2 fixed at model value of 0.0106. Largest monthly tides ~9 cm vertical amplitude.
- Tidal Q is 29±4 at 1 month. This is strong dissipation and may be due to a partial melt in the lower mantle.
- Q as a function of tidal period is $29(\text{Period}/27.212\ \text{d})^{0.07}$ so Q is about 35 at 1 yr.



Lunar Fluid Core

- Dissipation at the fluid-core/solid-mantle boundary (CMB) is strong. Core (outer) must be fluid.
- Inferred radius <355 km if core is iron (Yoder turbulent boundary layer theory). A lower density core could be larger.
- Product of core moment and CMB oblateness is $(3\pm1)\text{x}10^{-7}$. Fluid core effect.

Size of Forced Librations

- Largest longitude amplitudes: 90.7” at 1 yr, 16.8” at 3.0 yr, 16.8” at 27.555 d, 14.2” at 273 yr, 9.9” at 206 d, 7.8” at 18.6 yr, 6.8” at 6.0 yr, etc.
- Longitude resonance at 2.9 yr so forced periods span resonance giving resonance frequency.
- Nutation-like amplitudes (coordinates precess with node): 5553.6” mean tilt, 11.8” at 18.6 yr, 3.0” at 173 d, etc.
- Resonance at 24 yr.

Forced Librations

- Wobble-like (in rotating coordinates): 124.5x75.5” at 6.0 yr, 2.7x1.6” at 188 d, etc.
- Pole wobble resonance at 74.6 yr so largest forced terms are offset.
- Axes of rotation, spin, and body are well separated.
- 1” is 8.4 m at the lunar radius.

Physical Librations — History

- Based on early (triaxial) series, dissipation was expected to appear as phase shifts.
- Third-degree harmonics added terms with different phases complicating the possibility.
- Systematic signatures in range residuals (late 1970s) led Yoder to compute that the Cassini state would be phase shifted by dissipation.
- This 0.26” pole shift plus few mas dissipation effects allowed separation of tide and fluid-core/solid-mantle boundary dissipation (2001).

Free Librations

- One would expect free librations, the part of the rotation solution which depends on the initial conditions, to damp out, but all three mantle modes are observed for the Moon implying a stimulus.
- The longitude (2.9 yr) free libration is about 1.4” amplitude, but is confused by two forced terms.

More Free Librations

- The free wobble of the pole (analogous to Chandler wobble) has an elliptical 3.3”x8.2” path with a 74.6 yr period. This is 28x69 m for the lunar radius.
- The free precession mode with an 81 yr period is small at 0.02” amplitude.
- The fluid core mode is of unknown size.

Free Libration Damping

- Calculated damping times are about 25×10^3 yr for longitude, 2×10^6 yr for wobble (very uncertain), and 1.6×10^5 yr for free precession.
- Free librations cannot be primordial. Yoder suggests that eddies at the fluid-core/solid-mantle boundary stimulate wobble mode.

What Next?

- An inner (solid) core is likely, but not detected. It would add 3 more resonances to the physical libration response.
- Torques would include inner-core/mantle gravitational interactions.
- There are many few mm effects which need to be added to the overall model.

Summary

- Analysis of lunar ranges gives information on orbit, gravitational physics, geodesy, geophysics, and lunar science.
- Einstein's general relativity is confirmed. Earth rotation and station positions and motions are measured. Lunar tides are measured and fluid core is detected.

Summary Continued

- Lunar Laser Ranging continues to provide new results because of improving range and data analysis accuracies.
- The future offers improved accuracies and new results.